Collocation with Radial Basis Functions in a Pseudospectral Framework for Laminated Plates by the RMVT

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Motivation

Thin and thick **cross-ply laminated plates** require accurate prediction of static deformation and free vibration. Equivalent-single-layer (ESL) theories share degrees of freedom across layers, while layerwise (LW) theories assign layer-dependent DOFs and capture interlaminar effects.

- The **Carrera Unified Formulation (CUF)** expresses the governing equations of many plate theories through compact *fundamental nuclei*.
- The **Reissner Mixed Variational Theorem (RMVT)** treats displacements *and* transverse stresses as independent fields, enforcing *a priori* interlaminar continuity of transverse shear/normal stresses.
- We combine CUF + RMVT with an **RBF collocation** method in a **pseudospectral (PS)** framework, enabling generic geometries.

RBF–Pseudospectral collocation

The spatial part of the solution is a linear combination of smooth basis functions:

$$\hat{u}(x) = \sum_{j=1}^{N} c_j\,\phi_j(x), \qquad x \in \mathbb{R}.$$

Instead of polynomials (restricted to tensor-product grids), we use **radial basis functions**, allowing irregular grids without loss of accuracy. For the elliptic problem $\mathcal{L}u = f$ in Ω with $u = g$ on $\Gamma = \partial\Omega$, Kansa's unsymmetric collocation gives:

$$\hat{u}(\mathbf{x}) = \sum_{j=1}^{N} c_j\,\varphi(\|\mathbf{x} - \boldsymbol{\xi}_j\|), \qquad \mathbf{x} \in \Omega \subseteq \mathbb{R}^s,$$

with centers $\boldsymbol{\xi}_j$ and collocation points $\mathbf{x}_i$. The optimal shape parameter and differentiation matrices follow Ferreira & Fasshauer.

Governing equations by RMVT (fundamental nuclei)

For each layer k, expansion order τ, s:

$$\delta \boldsymbol{u}_s^{k\,T}: \; \boldsymbol{K}_{uu}^{k\tau s}\,\boldsymbol{u}_\tau^k + \boldsymbol{K}_{u\sigma}^{k\tau s}\,\boldsymbol{\sigma}_{n\tau}^k = \boldsymbol{P}_{u\tau}^k$$

$$\delta \boldsymbol{\sigma}_{ns}^{k\,T}: \; \boldsymbol{K}_{\sigma u}^{k\tau s}\,\boldsymbol{u}_\tau^k + \boldsymbol{K}_{\sigma\sigma}^{k\tau s}\,\boldsymbol{\sigma}_{n\tau}^k = \boldsymbol{0}$$

with natural boundary conditions imposed on displacements through the $\boldsymbol{\Pi}_u^{k\tau s}$, $\boldsymbol{\Pi}_\sigma^{k\tau s}$ operators.

Free vibration — fundamental frequency

Simply-supported $[0^\circ/90^\circ/90^\circ/0^\circ]$ square plate, $h/a = 0.2$, $\bar{w} = (wa^2/h)\sqrt{\rho/E_2}$.

Method	Grid	E_1/E_2			
		10	20	30	40
Liew		8.2924	9.5613	10.320	10.849
Exact (Khdeir)		8.2982	9.5671	10.326	10.854
Present	11×11	8.2866	9.5391	10.2676	10.7590
	13×13	8.2863	9.5388	10.2673	10.8035
	17×17	8.2862	9.5387	10.2672	10.8034

Conclusions

- A new **RBF–pseudospectral** formulation of the **RMVT** for laminated plates is presented, built on Carrera's Unified Formulation.
- Excellent agreement with elasticity and reference solutions for **transverse displacements**, **interlaminar stresses** and **free vibrations**, across all a/h ratios.
- Unlike FSDT, the method remains accurate for **thick laminates**, and RBFs allow **generic geometries** beyond tensor-product grids.

Selected references

- E. Carrera, *Apple. Mech. Rev.* (2003) — RMVT & CUF.
- A. J. M. Ferreira, G. E. Fasshauer, *CMAME* — RBF shape parameter & differentiation matrices.
- A. J. M. Ferreira et al., *RMVT + RBF collocation* (2013).
- N. J. Pagano; J. N. Reddy; K. M. Liew — benchmark solutions.

Static benchmark — cross-ply plate

Simply-supported $[0^\circ/90^\circ/90^\circ/0^\circ]$ square plate under sinusoidal load $p_z = P\sin(\pi x/a)\sin(\pi y/a)$. Material: $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = 0.25$. Normalized response:

$$\bar{w} = \frac{10^2\,w\,h^3 E_2}{Pa^4}, \quad \bar{\sigma}_{xx} = \frac{\sigma_{xx}h^2}{Pa^2}, \quad \bar{\tau}_{xz} = \frac{\tau_{xz}h}{Pa}.$$

a/h	Method	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\tau}_{zx}$
4	HSDT	1.8937	0.6651	0.6322	0.2064
	FSDT	1.7100	0.4059	0.5765	0.1398
	elasticity	1.954	0.720	0.666	0.270
	present 13×13	1.9784	0.6766	0.5872	0.2332
	present 17×17	1.9783	0.6766	0.5872	0.2332
	present 21×21	1.9783	0.6765	0.5872	0.2332
10	HSDT	0.7147	0.5456	0.3888	0.2640
	FSDT	0.6628	0.4989	0.3615	0.1667
	elasticity	0.743	0.559	0.403	0.301
	present 13×13	0.7326	0.5627	0.3909	0.3321
	present 17×17	0.7325	0.5627	0.3908	0.3321
	present 21×21	0.7325	0.5627	0.3908	0.3321
100	HSDT	0.4343	0.5387	0.2708	0.2897
	FSDT	0.4337	0.5382	0.2705	0.1780
	elasticity	0.4347	0.539	0.271	0.339
	present 13×13	0.4308	0.5432	0.2731	0.3774
	present 17×17	0.4307	0.5431	0.2730	0.3771
	present 21×21	0.4307	0.5431	0.2730	0.3768

Through-thickness stress distributions

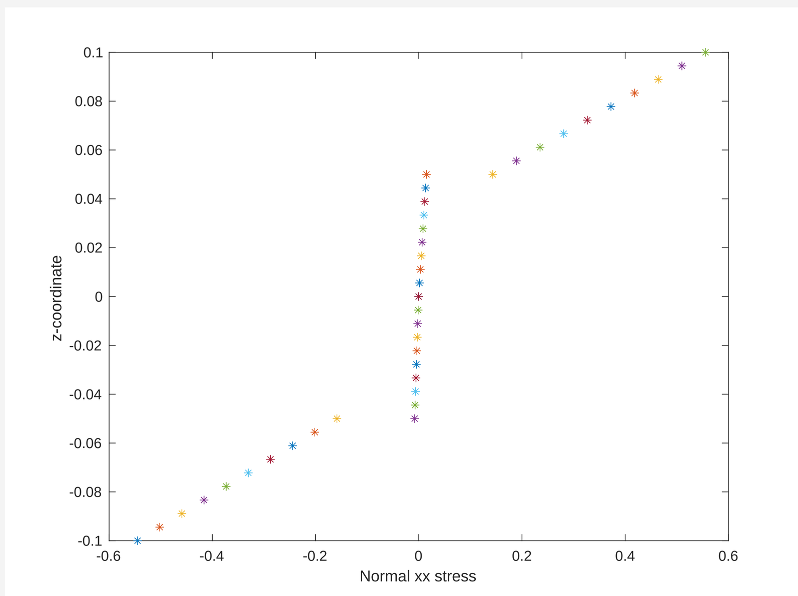

Fig. 1. σ_{xx}, $a/h = 10$, 21×21 pts

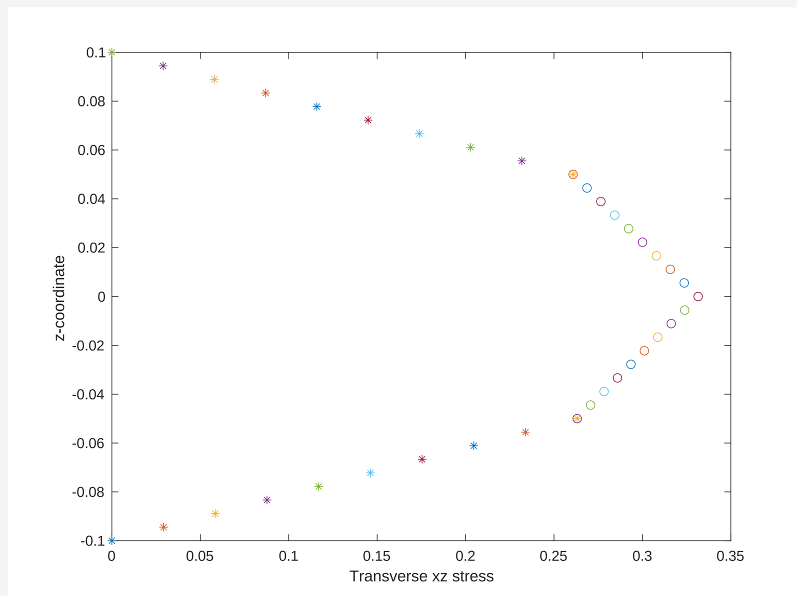

Fig. 2. τ_{xz}, $a/h = 10$, 21×21 pts

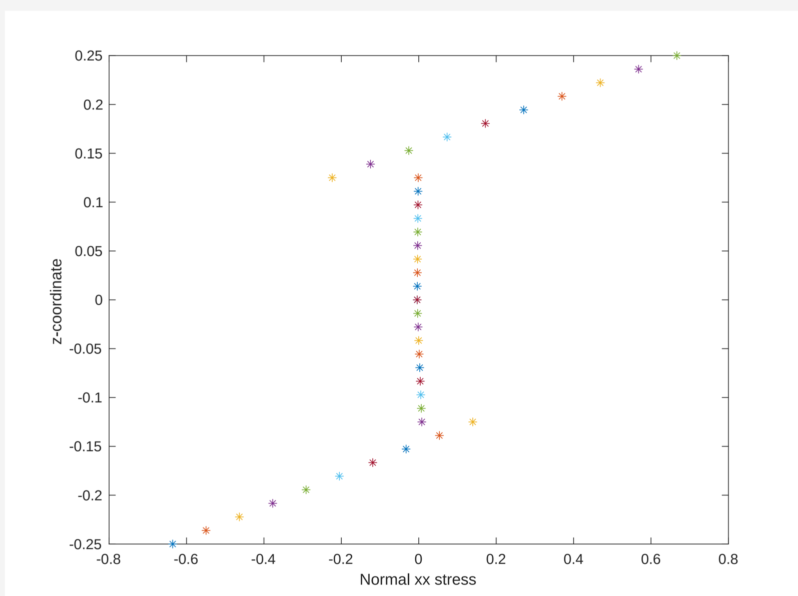

Fig. 3. σ_{xx}, $a/h = 4$, 21×21 pts

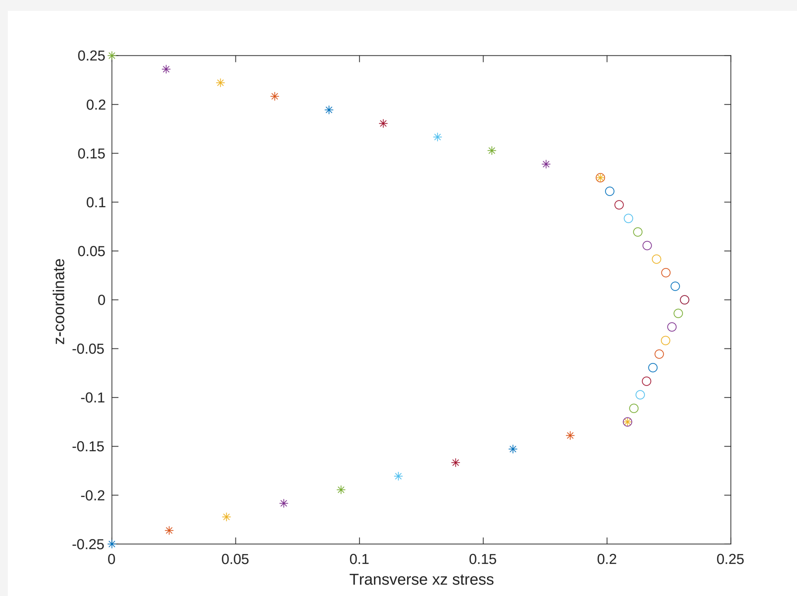

Fig. 4. τ_{xz}, $a/h = 4$, 21×21 pts

Transverse shear stresses at every interface are obtained directly from the constitutive equations.